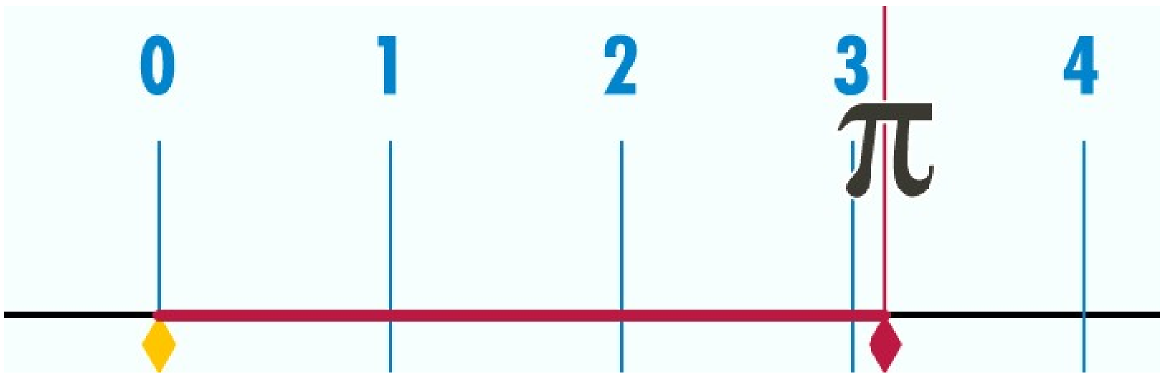
Trigonometric Ratios

Revision of concepts

We will now revise:

- What is π really?
- Why study trigonometry?
- What you know from lower classes:
  - Basic definitions
  - Basic identities
  - Trigonometric ratios for standard angles
- Measurement of Angles
- Circular function definition of trigonometric functions.
- Sign of trigonometric functions in different quadrants
- Generating graphs of sinx and cosx.
- Graphs of six basic trigonometric functions and their properties.
- Basic graphical transformations
- Trigonometric ratios of Allied angles

What is π really?



π is an irrational number that represents the ratio of a circle's circumference to its diameter.

π = 3.1415926535 8979323846 2643383279 ...

For approximation use: $\pi \sim 3.1416$ (given by Aryabhatta) or $\frac{22}{7}$

Why study trigonometry

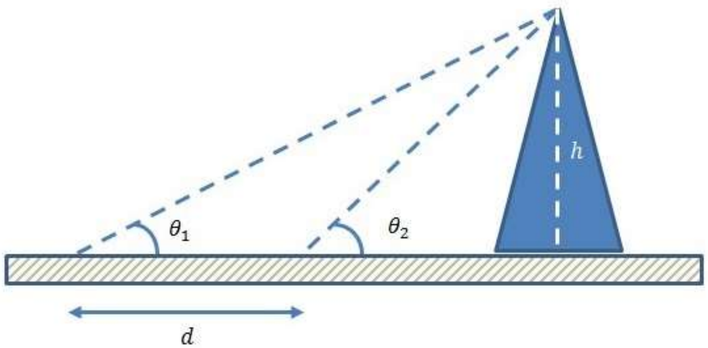
Applications in fields such as

- Astronomy
- Architecture
- Electrical wiring and electronic circuit design
- Functioning of sense organs
- Map making and navigation
- Physics of music (sound waves)

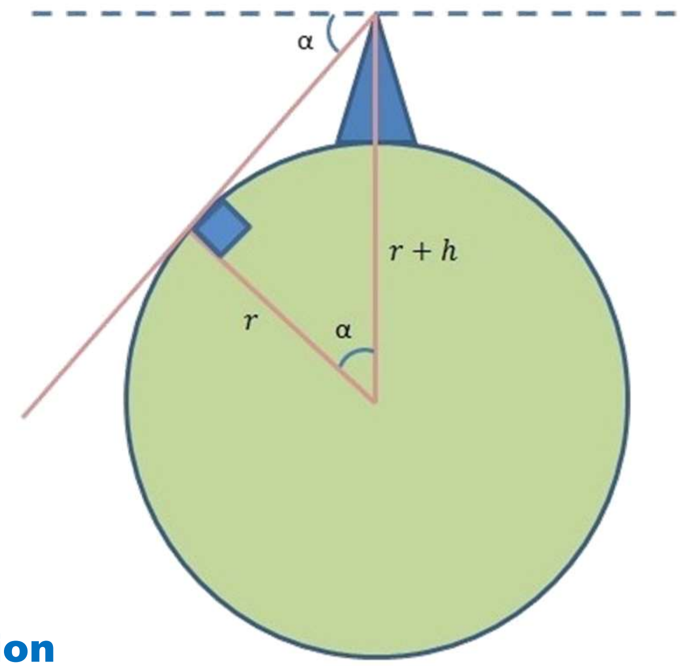
And many more…

Let's look at some basic examples

Astronomy:
Al Beruni and radius of earth



$$h = \frac{d\,tan\theta_1 tan\theta_2}{tan\theta_2 - tan\theta_1}$$



$$r = (r + h)cos\alpha$$

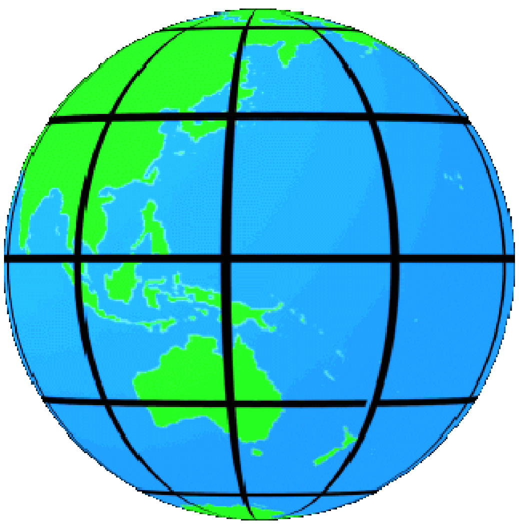
$$r = \frac{h cos\alpha}{1 - cos\alpha}$$

Step 1 : Measure the height of a hill by measuring angle of elevation from two different positions.

Step 2: Measure the angle of dip of the horizon from top of that hill.

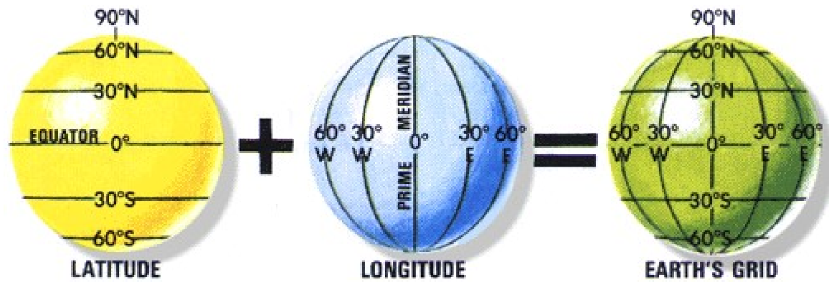
Was done around yr 1000 AD and was 99% accurate

Navigation, map-making, exploration

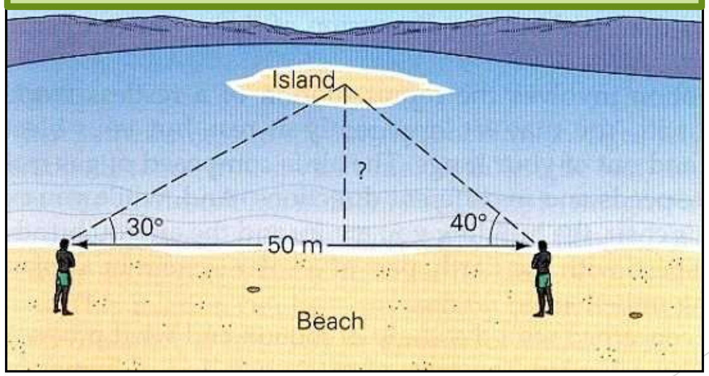


Longitudes and latitudes are basically navigation coordinates based on angles

Navigation

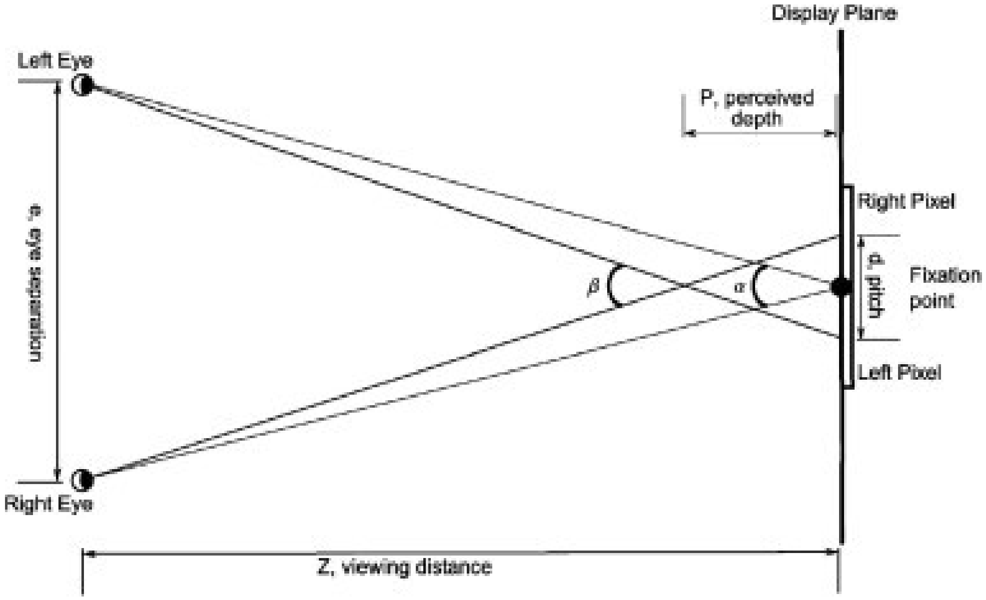


Exploration



Can you estimate the unknown distance here?

Depth perception by our eyes and ears



Differing angle of seeing an object allows the brain to estimate its distance, giving us the ability to see in 3D.

What you learnt in lower classes:

Basic definitions:

(i) sin θ = $\frac{p}{h}$

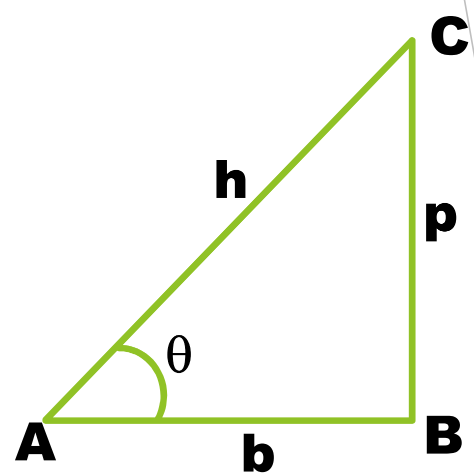
(ii) cos θ = $\frac{b}{h}$

(iii) tan θ = $\frac{p}{b}$

(iv) cot θ = $\frac{b}{p}$

(v) sec θ = $\frac{h}{b}$

(vi) cosec θ = $\frac{h}{p}$



What you learnt in lower classes:

Basic identities:

(i) $\sin^2\theta + \cos^2\theta = 1$

(ii) $\sec^2\theta - \tan^2\theta = 1$

(iii) $\text{cosec}^2\theta - \cot^2\theta = 1$

What you learnt in lower classes:

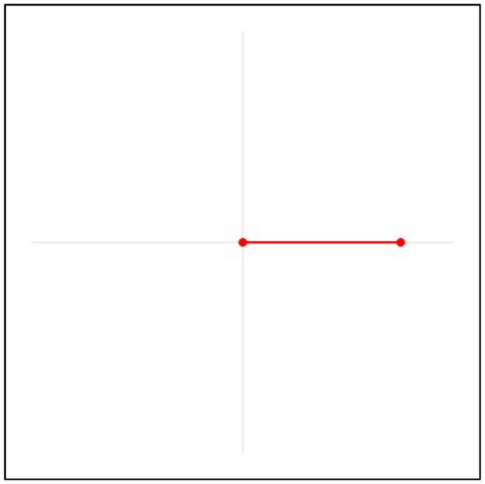
Trigonometric ratios for standard angles:

T-ratio \ Θ	0°	30°	45°	60°	90°
sin Θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos Θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan Θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec Θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec Θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot Θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Measurement of angles

In lower classes we measured angles in degrees.
In higher classes we measure angles in radians as well as degrees.

But what is a radian really?



One radian is the angle subtended at the center of a circle by an arc that is equal in length to the radius of the circle.

You may verify from the diagram that a complete circle has 2π radians.

Converting from radians to degrees and vice versa

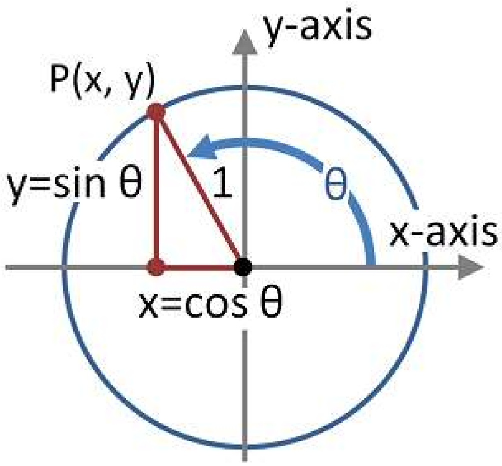
Radians to degrees:

$$x^c = \left(x\,\frac{180}{\pi}\right)^o$$

Degrees to Radians:

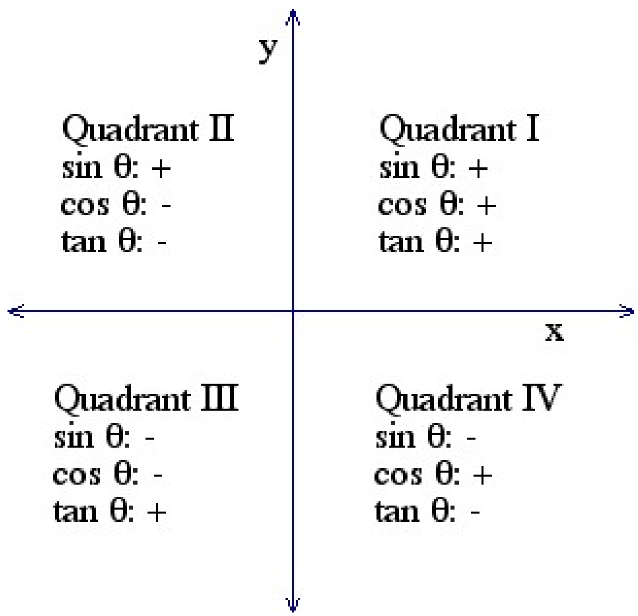
$$\theta^o = \left(\theta\,\frac{\pi}{180}\right)^c$$

Defining trigonometric functions as circular functions.

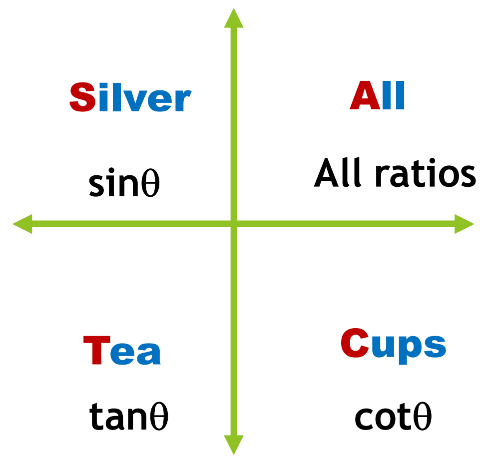


For a point P moving on a unit circle centered at origin, O, when OP makes angle θ with X-axis (taken anti-clockwise) then coordinates of P are (sinθ, cosθ)

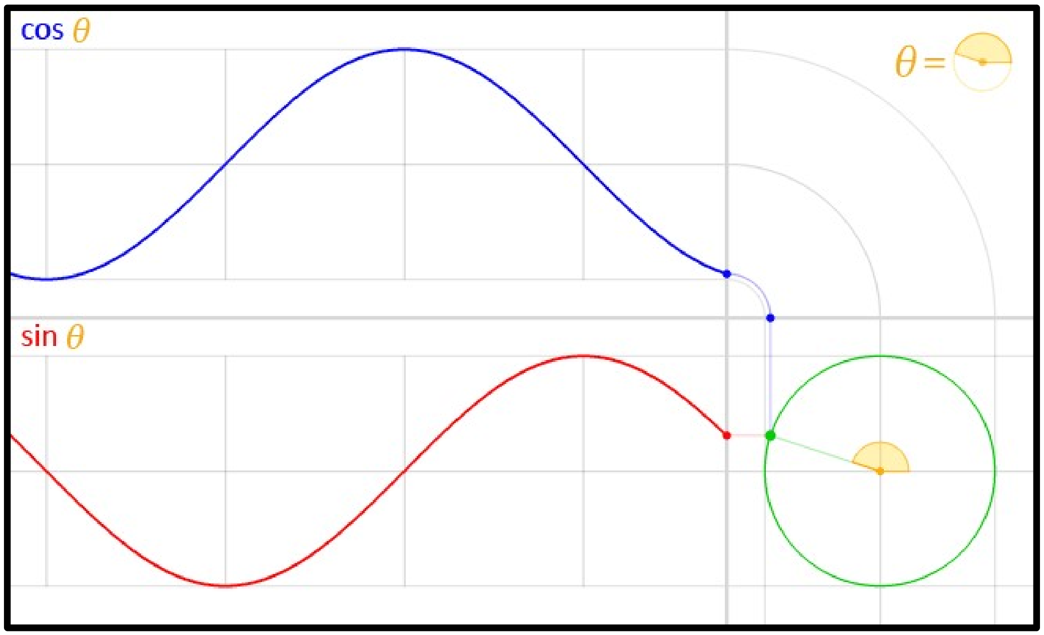
Sign of trigonometric functions in different quadrants



Mneumonic to remember



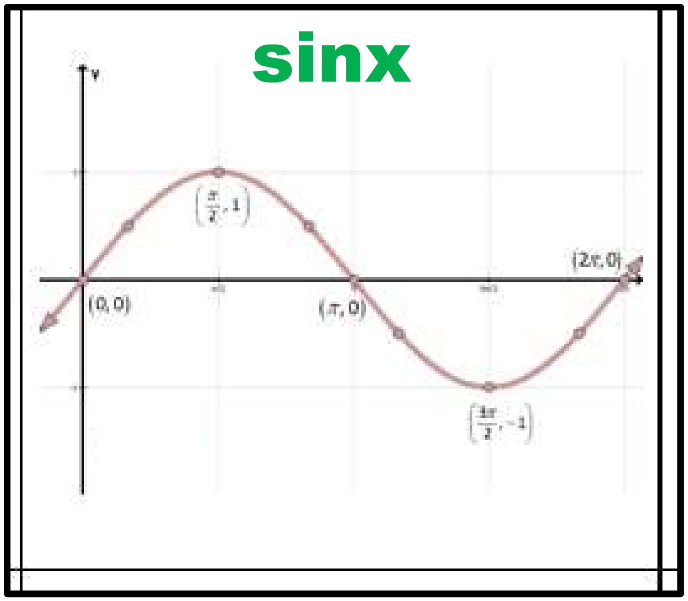
Generating the graphs of sine and cosine functions

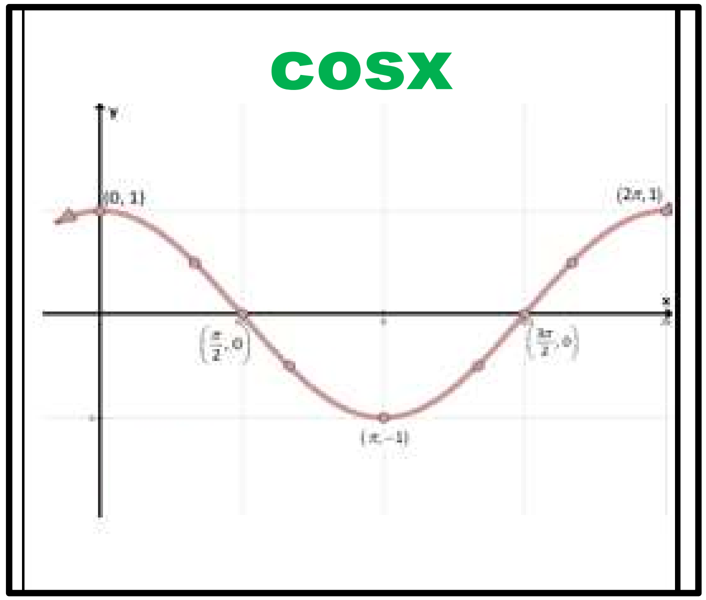


The coordinates of the green dot are

(cos θ, sin θ)

Graphs of sinx and cosx





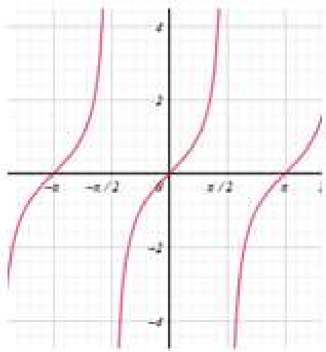
Observe that both sin x and cos x satisfy

$-1 \leq \sin x \leq 1$

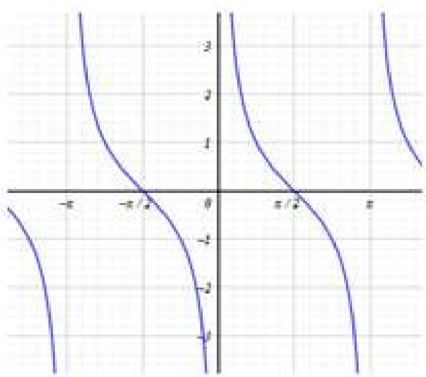
$-1 \leq \cos x \leq 1$

Graphs of tanx and cotx

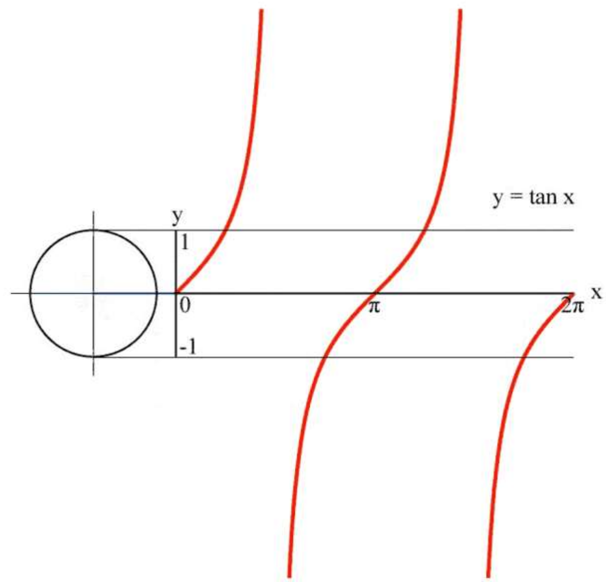
tan x



cot x

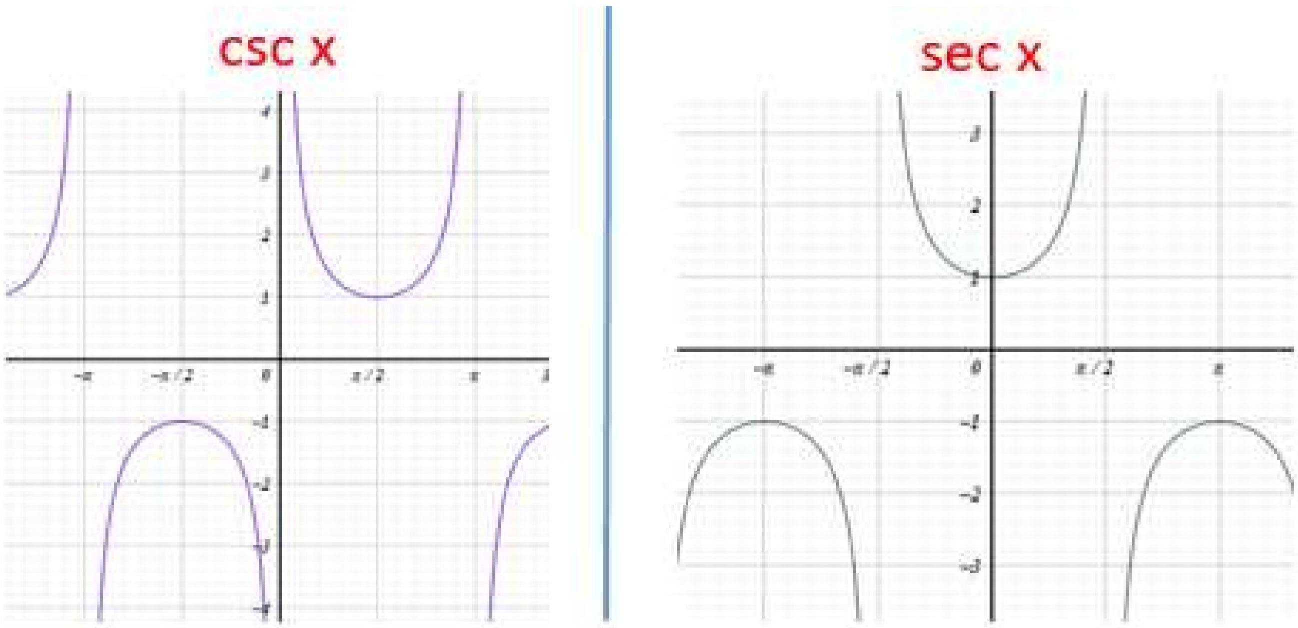


Generation of y = tanx from the circular definition.



Observe that tanx ∈ $(-\infty, \infty)$ and cotx ∈ $(-\infty, \infty)$

Graphs of sec x and cosec x

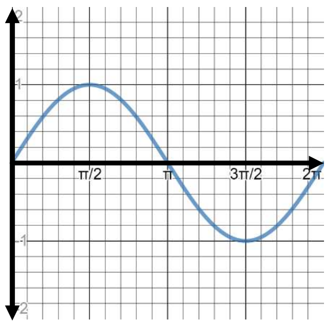


Observe that both functions satisfy:
|cosec x| ≥ 1 and |sec x| ≥ 1

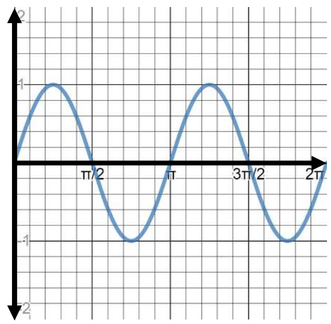
i.e. cosec x or sec x ≥ 1 or ≤ -1.

Basic graphical transformations

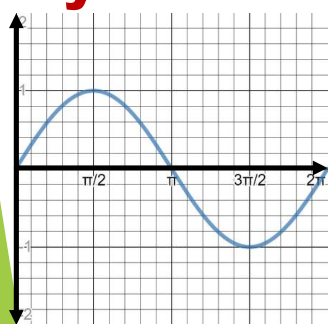
$y = \sin x$



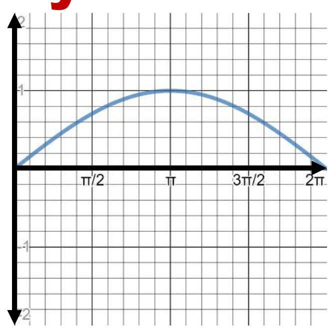
$y = \sin 2x$



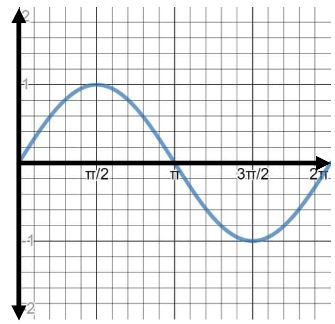
$y = \sin x$



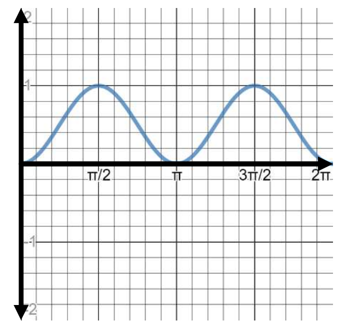
$y = \sin x/2$



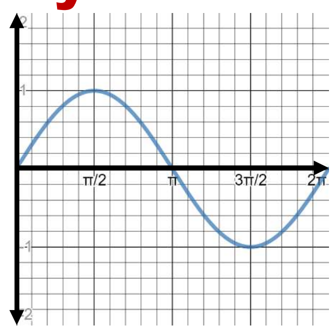
$y = \sin x$



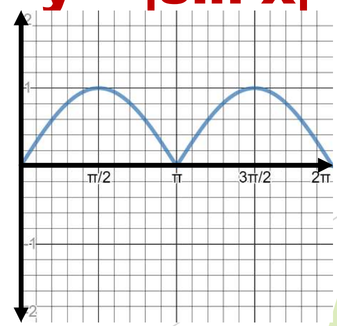
$y = \sin^2 x$



$y = \sin x$



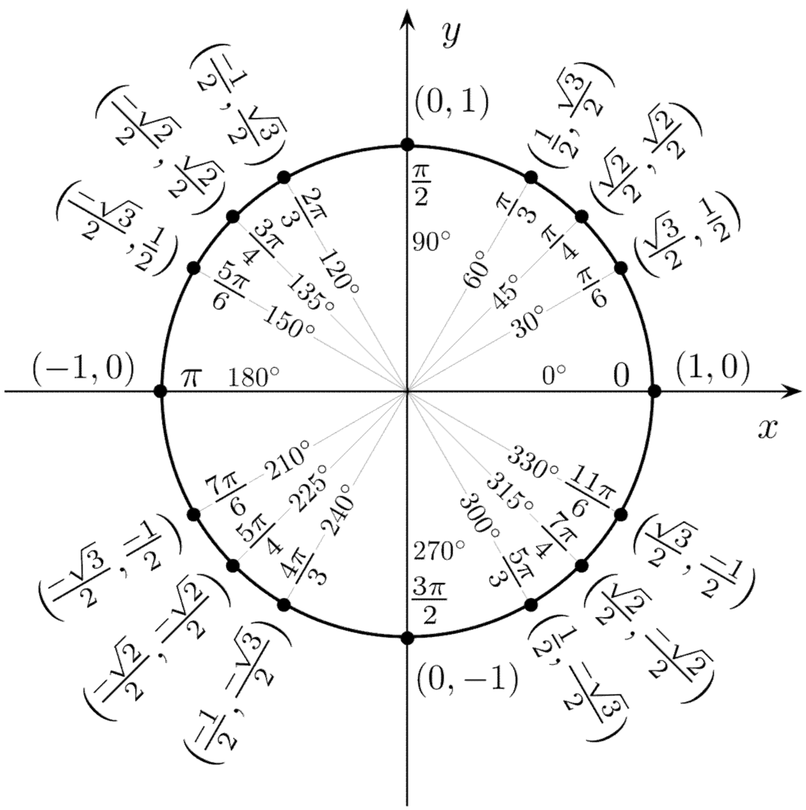
$y = |\sin x|$



Trigonometric ratios of allied angles

Allied angle	Formula
$-\theta$	$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$
$90^\circ - \theta$	$\sin(90^\circ - \theta) = \cos\theta$ $\cos(90^\circ - \theta) = \sin\theta$
$90^\circ + \theta$	$\sin(90^\circ + \theta) = \cos\theta$ $\cos(90^\circ + \theta) = -\sin\theta$
$180^\circ - \theta$	$\sin(180^\circ - \theta) = \sin\theta$ $\cos(180^\circ - \theta) = -\cos\theta$
$180^\circ + \theta$	$\sin(180^\circ + \theta) = -\sin\theta$ $\cos(180^\circ + \theta) = -\cos\theta$

Representation of standard ratios as coordinates



Coordinates shown here are (cos θ, sin θ)

Thank you ☺